Endogenous adaptive dynamics in Pasinetti model of structural change

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Abstract: This paper formalises, in a very stylised way, localised learning of consumers and of producers within Pasinetti model of structural change (Pasinetti (1965, 1981, 1993)). Unlike Pasinetti model and other models of structural change, which either consider technological change exogenously given (see e.g. Baumol (1967), Pasinetti (1965, 1981, 1993), Araujo, R.A., Texeira, J.R. (2002)) or assume perfect rationality (see, e.g. Laitner (2001), Kogensmut, Rebelo et (2201)), in this paper we endogenise technological and consumption dynamics by assuming bounded rational firms and consumers. We provide a concept of (secular) equilibrium and study the dynamic properties of the economy. We show also that the theoretical framework here provided is able not only to deal with improvements in methods of production, but it is also able to deal with product innovation as well. Moreover, our model can easily and consistently incorporate a variety of firms, this allowing the study of the dynamics of the economy from an evolutionary point if view (see e.g. Metcalfe (1995), (1998)). Therefore, our model could be interpreted as providing also a bridge between the literature on structural change and that one on evolutionary dynamics.

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"We must recognize that knowledge of the extent of production possibilities, and of the means and paces of their enlargement, is gained only through experience in their use and extension. Optimization and exploration thus have to be engaged in simultaneously, with the latter serving as guide and strengthen the former. The problem takes on some of the aspects of the ascent of a mountain wrapped in fog. Rather than searching for a largely invisible optimal path, one may have to look for a good rule for choosing the next stretch of the path with the help of all information available at the time" T. C. Koopmans (1967, p. 12)

1. Introduction

An increasing dissatisfaction, both at theoretical and at empirical level, is emerging as far as neoclassical disaggregate growth theory is concerned. In fact, either the usual hypotheses adopted by this theory, i.e. balanced growth and perfect foresight by agents, are evidently implausible. Although these assumptions have not been accepted unanimously by neoclassical economists - as the Koopmans' opinion quoted at the beginning attests - they are customarily and uncritically used by neoclassical economists and by now characterise their analysis.

However, a growing theoretical as well as empirical interest is emerging for the study of economies in which these assumptions are relaxed. On the one hand, apart the by now "classical" analysis of Baumol (1967), Pasinetti (1964), (1981), (1993), Carter (1970) and Leon (1967), quite a few works have been done recently at empirical and theoretical level for incorporating structural change in the analysis of dynamic economies (Baumol, Blackman and Wolff (1985), Baumol and Wolff (1995), Cornwall and Cornwall (1994), Reati and Raganelli (1993), Gundlach (1994), Punzo (1995), Reati (1995), Notarangelo (1999), Araujo and Texeira (2002), Montobbio (2002), Lavezzi (2003)). Focussing our attention only on the theoretical works, the main aim of these works has been to incorporate sectors growing at different rates of growth and to study the economic implication of these differences. However, all these works share the common unsatisfactory feature of assuming the dynamics of technology and demand determined by exogenously given rules. An important consequence of this assumption is that these works are not able to *explain* not only process innovation but also product innovation, i.e. changes over time in the number of commodities produced.[1] Clearly, any model which pretends to explain satisfactorily the dynamics of modern economies has to overcome such shortcomings. More specifically, any *theoretically* adequate model of structural dynamics has to explain the growth (or decay) of sectors and their technological dynamics in terms of process *and* product innovations as the outcome of rational *decisions* by economic agents. However, on the other hand and following Koopmans' suggestion, any *empirically* acceptable model should not suppose too much rationality by agents, as it is widely held that agents take decision on the basis of imperfect knowledge about the environment and with limited capacity of elaborating information. Moreover, since the dynamics is generated mainly by human learning (see e.g. Pasinetti (1993), Romer (1990) Aghion and Howitt (1998)), an explicit treatment of the growth of learning should be explicitly incorporated.

The awareness of the literature on structural change as far as the problem of learning is concerned makes this literature very close to evolutionary theory where this problem is notoriously one of the focal point of analysis (see Nelson and Winter (1981) and for surveys Dosi and Nelson (1994), Dosi and Winter (2002)). Apart the central role of lack of perfect knowledge of the environment in which agents operate and the view that the improvement of knowledge is an objective actively pursued by them, further important features of the evolutionary approach are the heterogeneity of agents (consumers and firms), the persistence of their variety over time, and the evolution of the economy through the selection process operated usually via the market mechanism(see e.g. Dosi and Winter (2002, p. 387-388)). These aspects have been recently tackled by Montobbio (2002), who develops a general evolutionary model of structural change. However, his model is still a partial equilibrium model as interdependence between sectors is originated via the demand side only. Finally, one should not forget that one of the aim of evolutionary theory is to propose an alternative theory of growth and dynamics to neoclassical economics, and from this point of view evolutionary growth theory and the theory structural change seem to be complementary (see Metcalfe et al. (2005)).

[1] Models like Pasinetti's one deal more or less explicitly with changes in the number of commodities; however, no formal analysis is provided as far as this phenomenon is concerned (see, for example, Pasinetti (1981, p. 206 ff.))

The aim of this paper is to develop a dynamic multisectoral model with structural change *à la* Pasinetti model in which structural change is the outcome of an adaptive economising behaviour of consumers and firms with limited knowledge. Following Pasinetti, the analytical framework we use is a linear model of production *à la* Sraffa-Leontief, in which normal prices[2] and gross output evolves over time because of changes in technology and in final demand vector. The evolution of technology is due to localised growth of knowledge of producers, while changes in the final demand is generated by localised growth of knowledge of households in terms of preferences and feasible consumption sets. All agents are assumed to take decisions in an adaptive way, i.e. on the basis of their current set of information and preferences that are determined by past experience. The formalisation of dynamics of technology and households' knowledge is very close to the one envisaged by the evolutionist literature (see e.g. Cimoli and Dosi (1995, Section 2)) and has a strong empirical ring, it being close to the view endorsed by Evolutionary Operation literature (see Box (1957)) and to the idea of "adaptive economising" developed over a long span of years by Richard H. Day (see e.g. Day and Kennedy (1971), Day and Groves (1975), Day and Singh (1977), Day and Cigno (1978), Benhabib and Day (1981), Day (2002)). As a matter of fact, this approach is considered as a possible search strategy by students of the cognitive aspects of research and problem-solving (see e.g. Perkins and Weber (1992)) and it seems to be employed systematically in specific industries and in R&D activity (see for example, Vincenti (2000), Carlson (2000))

The paper is organised as follows. In the next section we provide the basic static model, introduce the main terminology and the basic assumptions. In Section 3 we shall introduce a concept of stationary equilibrium similar to the Marshallian concept of secular equilibrium (see Marshall (1890, Chapter V)) and study the dynamic properties of this model. Section 4 will provide a

[2] Actually, Pasinetti initially interprets prices as short-period prices and changes of techniques and of demand as long-period phenomena (see Pasinetti, 1981, p. 29). This is consistent with his assumption that rates of profit are different across sectors. However, in developing his analysis Pasinetti eventually points out that the rates of profits should be taken as equal across sectors (see Pasinetti, 1981, p. 151), hence, prices should be normal prices. We prefer to stick at the beginning with the Marshallian view according to which production prices (or normal prices) are determined in the long run and changes of them "caused by the gradual growth of knowledge, …, and the changing conditions of demand ad supply from one generation to another" (Marshall, 1890, p. 315) belongs to the realm of secular movements. See, however, Section 2 for possible alternative pricing rules.

numerical example showing that our model is able not only to consider process innovations but it is able to deal also with changes over time in the number of commodities produced. Finally, following the view supported by evolutionary economics, according to which even in the long run firms can differ from the technological point of view because the tacit nature of knowledge (see e.g. Cimoli and Dosi (1995, p. 248)), in Section 5 we briefly indicate the way to extend our model to allow heterogeneous firms in the same industry in order to show that our model can easily and consistently be developed within the evolutionary theory (see, e.g. Metcalfe (1998), (1996)). Such a model can be considered as the extension to a fully-fledged multisectoral model of Montobbio's work.

2. The basic multisectoral model

2.1. The production side.

Consider a single production n good economy *à la* Sraffa (1960). We suppose that the set of all possible techniques potentially available in sector $i = 1, 2, \ldots, n$ is represented by set X_i. Set $X = \Pi_i X_i$. A generic process of sector i is denoted by $\mathbf{b}_i = (\mathbf{a}_i, l_i)$, where $\mathbf{a}_i$ is the n-dimensional input vector and l_i is the labour coefficient. A generic technique is described by matrix $\mathbf{A} = (\mathbf{a}_1, \mathbf{a}_2, \ldots, \mathbf{a}_n)^{\mathrm{T}}$ of input coefficients and by a vector $\mathbf{l} = (\mathrm{l}_1, \mathrm{l}_2, \ldots, \mathrm{l}_n)$ of labour coefficients, where $\mathbf{b}_i = (\mathbf{a}_i, l_i) \in X_i$ (the superscript T indicates the transposition operator). A generic technique $(\mathbf{A}, \mathbf{l}) \in X$ is indicated also by $\mathbf{T}$.

For the sake of determinatess, we assume that if technique $\mathbf{T} = (\mathbf{A}, \mathbf{l})$ is used, then the following production price equation is associated: $(1+r(\mathbf{T}))\mathbf{A}\mathbf{p}(\mathbf{T}) + w(\mathbf{T})\mathbf{l} = \mathbf{p}(\mathbf{T})$, with the usual meaning of symbols. The economic meaning of the production prices previously introduced is that free competition among industries equalises the rate of profit over industries. This approach is rooted in classical tradition and it is characterised by the use of the long period method of analysis (see, e.g. Pasinetti (1975), Kurz and Salvadori (1995), Foley and Michl (1999)) and is in line with Pasinetti's analysis of structural change (see footnote 2). It is worth pointing out that, however, this is just one way to determine prices from technology, and alternatives, equally theoretically and/or empirically appealing ways to do it can be envisaged. For example, prices can be determined by following the "full-cost" approach (see, e.g. Hall and Hitch (1939), Marris (1964), widely employed by the post-keynesian theory.[3] By the way, the extension of the latter work within the structural change context would make a bridge between structural change and post-keynesian theory.

Assumption 1. For every $i = 1,\ldots, n$, set X_i is a compact subset of $\Re_+^{n+1}$; moreover, for any $\mathbf{b}_i \in X_i$, one has that $\mathbf{a}_i \geq 0$ and $\mathbf{a}_i \neq 0$. Finally, for every i, $l_i > 0$.

Assumption 2: For every $\mathbf{T} = (\mathbf{A}, \mathbf{l}) \in X$, the dominant eigenvalue of matrix $\mathbf{A}$ is less then 1. Moreover, $\mathbf{A}$ is indecomposable.

Assumption 3. The amount of labour available is fixed al level $L > 0$.

Compactness of sets X_i is a simplifying assumption. The last condition of Assumption 1 means that labour is necessary to produce any commodity. Assumption 2 ensures that any technique available is productive and that all goods are "basics", i.e. are used directly or indirectly in producing all commodities in the economy (see Sraffa (1960)); the last condition is a very stringent requirement. The reason for this assumption is mainly technical (see the proofs of Lemma 2 and Proposition 1). Assumption 3 is adopted here just for the sake of simplicity and it allows us to focus on the dynamics generated by consumption and technical progress only rather than the dynamics generated by the increase in population.

We assume that for every $\mathbf{T} \in X$, $\mathbf{d}^{\mathrm{T}}\mathbf{p}(\mathbf{T}) = 1$, with $\mathbf{d} \in \Re_+^n$; i.e. the bundle of commodity $\mathbf{d}$ is used as *numeraire.*

[3] For a multisectoral model of balanced growth with full cost pricing see D'Agata (2006).

Lemma 1. The following assertions hold true:

(i) If $r(\mathbf{T}) = 0$ for every $\mathbf{T} \in X$, then $\mathbf{p}(\mathbf{T})$ and $w(\mathbf{T})$ are continuous and strictly positive functions of $\mathbf{T}$.

(ii) There exists a strictly positive number w^* such that if $w = w^*$, then $\mathbf{p}(\mathbf{T})$ and $r(\mathbf{T})$ are continuous and strictly positive functions of $\mathbf{T}$.

Proof. (i) Immediate from Assumpions 1 and 2.

(ii) Set $r(\mathbf{T}) = 0$ and define $0 < w^* < \min\,[w(\mathbf{T}) \mid \mathbf{T} \in X]$. The assertion follows from the fact that set X is compact and from point (i) before.

From now on we assume that $w = w^*$.

Suppose that technique $\mathbf{T} = (\mathbf{A}, \mathbf{l}) \in X$ is in use and $\mathbf{P}(\mathbf{T}) = (\mathbf{p}(\mathbf{T}), r(\mathbf{T}))$ are the associated price vector and rate of profit. We say that method $\mathbf{b}_i^{'} = (a_i^{'}, l_i^{'}) \in X_i$ pays positive extra-profits at the current production prices $\mathbf{P}(\mathbf{T})$ if $(1 + r(\mathbf{T}))\mathbf{a}_i^{'}\mathbf{p} + w\mathbf{l}_i^{'} < p_i$.

Producers' behavioural assumption. Producers introduce a new method if and only if it pays positive extra-profits at the current production prices.

The following lemma, whose proof follows from indecomposability of matrices $\mathbf{A}$, yields that the uniform rate of profit does increase whenever a new method paying positive extra-profits is introduced.

Lemma 2. If $(1+r(\mathbf{T}))\mathbf{A}'\mathbf{p}(\mathbf{T}) + w\mathbf{l}' \leq \mathbf{p}(\mathbf{T})$ with a strict inequality satisfied for at least one industry, then $r(\mathbf{T}') > r(\mathbf{T})$.

2.2. The consumption side.

To each technique $\mathbf{T} = (\mathbf{A}, \mathbf{l}) \in X$ is associated also a quantity system: $\mathbf{xA} + \mathbf{y} = \mathbf{x}$, where $\mathbf{y}$ is the net production vector. Let $Q(\mathbf{T}) = \{\mathbf{y} \in \Re_+^n \mid \mathbf{xA} + \mathbf{y} = \mathbf{x},\ \mathbf{x} \in \Re_+^n,\ \mathbf{xl} \leq L\}$ be the set of feasible net production vectors. Under Assumption 1, the validity of Lemma 3 is immediate.

Lemma 3. The gross production vector $\mathbf{x}$ solution to the quantity system $\mathbf{xA} + \mathbf{y} = \mathbf{x}$ is a continuous function of the net consumption vector $\mathbf{y}$ and of the technology $\mathbf{T} = (\mathbf{A}, \mathbf{l})$.

Lemma 4. For every $\mathbf{T}\in X$, set $Q(\mathbf{T})$ is compact. Moreover, $Q(X) = \bigcup_{\mathbf{T}\in X} Q(\mathbf{T})$ is compact as well.

Proof. The former statement follows from Assumption 1. The latter statement is ensured if we show that the correspondence defined by set $Q(\mathbf{T})$ is upper hemi continuous (see, e.g. Aliprantis and Border (2006, p.560)). However, correspondence Q is actually continuous as can routinely be proved by using the sequential characterization of upper and lower hemi continuity (Aliprantis and Border (2006, p. 563 ff.)).

We model the behaviour of households as consumers with preferences defined only on a *subset B* of the net production space $\Re^n_+$. The subset on which preferences are defined can be interpreted as an abstract version of the opportunity set *à la* Koopmans (1964), and it can be justified by the fact that defining preferences is a costly (or, at least, a time-consuming) activity; hence, households have no possibility or advantage to express a complete system of preferences over the whole possible net production set. Thus, let us denote by $\succsim_B$ the (aggregate) preference relation of households with respect to the subset B of net production vectors. We adopt the following assumption:

Assumption 3. For every $B \subseteq \Re^n_+$ the preference system $\succsim_B$ satisfies the conditions of completeness, transitivity and continuity.

Notice that Assumption 3 allows the preference system $\succsim_B$ to change as the subset B changes. This is in line with the phenomenon of "context dependent preferences", which is widely accepted by the economic literature (see, e.g. Sen (1997)). The following behavioural assumption is also worth emphasising:

Consumers' behavioural assumption. For every $\mathbf{T}\in X$, households choose the aggregate net consumption vector $\mathbf{y}$ in set $Q(\mathbf{T})\cap B$, according to their preferences $\succsim_B$.

3. Adaptive multisectoral model with structural change

Consider the economy introduced in the previous section and suppose that the firms' technology knowledge in industry i grows over time according to a rule defined by the *correspondences*: Φ_i: $X \times \Re^n_+ \times \Re^{n+1}_+ \rightarrow\rightarrow X_i$, indicated by $\Phi_i(\mathbf{T}, \mathbf{x}, \mathbf{P})$, while the consumption knowledge of households grows over time according to the *correspondence* $\Gamma: \Re^n_+ \times \Re^{n+1}_+ \rightarrow\rightarrow \Re^n_+$, indicated by $\Gamma(\mathbf{y}, \mathbf{P})$. More specifically, suppose that at time t technique $\mathbf{T}_t = (\mathbf{A}_t, \mathbf{l}_t)$ is employed and the net demand vector is vector $\mathbf{y}_t \in Q(\mathbf{T}_t)$. From the arguments in the previous section, it follows that from $\mathbf{T}_t$ and $\mathbf{y}_t$ we obtain a price vector $\mathbf{P}(\mathbf{T}_t)= (\mathbf{p}(\mathbf{T}_t), r(\mathbf{T}_t))$ and a gross production vector $\mathbf{x}(\mathbf{T}_t, \mathbf{y}_t)$. According to the previous interpretation, set $\Phi_i(\mathbf{T}_t, \mathbf{x}(\mathbf{T}_t, \mathbf{y}_t), \mathbf{P}(\mathbf{T}_t))$ denotes the set of techniques "discovered" in industry i at time t and available at time t+1, while set $\Gamma(\mathbf{y}_t, \mathbf{P}(\mathbf{T}_t))$ denotes the set of net production bundles "explored" by households at time t and over which they are able to express at time t+1 preferences satisfying Assumption 3. As already said, the idea behind the households' behaviour is that the formation of preferences is a costly activity, both in psychological terms and in economic terms. Hence, at each time households carry out a local "exploration" of the net consumption space and are able to express "nice" preferences only for the known subset so obtained. This subset, which is defined by the correspondence Γ, is assumed to depend upon the current consumption choice and upon prices. As far as firms are concerned, correspondences Φ_i would capture the idea, widely accepted by the orthodox and non-orthodox economic literature, of local "discovery" of methods of production (see, for example, Atkinson and Stiglitz (1965), Antonelli (1995)). The increase in the set of technological opportunities is assumed to depend upon the current technique, the gross production vector and the price vector. It is worth to emphasise that our analysis is able to explain localised technological progress via learning by doing, R&D activity and technological spillovers among sectors as well.

The following assumptions will be adopted:

Assumption 4. For every i, correspondence Φ_i is non-empty, compact valued and continuous. Finally, for every $\mathbf{T} \in X$, $\mathbf{x} \in \Re^n_+$, $\mathbf{P} \in \Re^{n+1}_+$, $\mathbf{T} \in \Phi_i(\mathbf{T}, \mathbf{x}, \mathbf{P})$.

Assumption 5. Correspondence Γ is non-empty, compact valued and continuous. Moreover, for every $\mathbf{T} \in X$, $\mathbf{y} \in \Re^n_+$ and $\mathbf{P} \in \Re^{n+1}_+$, $\mathbf{y} \in \Gamma(\mathbf{y}, \mathbf{P})$ and $\Gamma(\mathbf{y}, \mathbf{P}) \cap Q(\mathbf{T}) \neq \varnothing$.

The first part of Assumptions 4 and 5 are technical and intuitively they mean that the set of techniques discovered by firms and of bundles of net production vectors on which households express their preferences change "smoothly" with respect to the relevant variables. The inclusion part of these assumptions mean that the "discovery" of new technological opportunities for firms and the set of commodity bundles on which households are able to express their preferences occur "around" the current choice (technique or consumption vector). This is a quite natural assumption to adopt. Finally, the non-emptiness condition in Assumption 5 is required to ensure that households are able to make feasible choices.

Set $F_i(\mathbf{T}, \mathbf{y}) = \Phi_i(\mathbf{T}, \mathbf{x}(\mathbf{T}, \mathbf{y}), \mathbf{P}(\mathbf{T}))$, $G(\mathbf{T}, \mathbf{y}) = \Gamma(\mathbf{y}, \mathbf{P}(\mathbf{T}))$ and $H(\mathbf{T}, \mathbf{y}) = G(\mathbf{y}, \mathbf{P}) \cap Q(\mathbf{T})$. The following lemma points out that the properties of Φ_i and Γ given in Assumptions 4 and 5 are transferred to correspondence F_i and G.

Lemma 5. The following assertions hold true:

(i) For every i, correspondence F_i is non-empty, compact valued and continuous. Finally, for every $\mathbf{T} \in X$ and $\mathbf{y} \in \Re^n_+$, $\mathbf{T} \in F_i(\mathbf{T}, \mathbf{y})$;

(ii) Correspondence G is non-empty, compact valued and continuous. Moreover, for every $\mathbf{T} \in X$, and $\mathbf{y} \in \Re^n_+$, $\mathbf{y} \in G(\mathbf{T}, \mathbf{y})$ and $H(\mathbf{T}, \mathbf{y}) \neq \varnothing$.

Proof. Non-emptiness is obvious. Compact valuedness and continuity follows from continuity of the price and the gross production vectors (see Lemmas 1 and 3) and from well known properties of composition of correspondences (see Aliprantis and Border (2006, p. 566)). The remaining properties are immediate consequences of Assumptions 4 and 5.

A (*local*) *secular configuration* (henceforth, a LSC) is a couple: ($\mathbf{T}^*$, $\mathbf{y}^*$) which satisfies the following conditions :

(i) $\mathbf{T}^* \in F(\mathbf{T}^*, \mathbf{y}^*)$,

(ii) $\mathbf{y}^* \in H(\mathbf{T}^*, \mathbf{y}^*)$,

(iii) $(1+r(\mathbf{T}^*)\mathbf{a}_i p(\mathbf{T}^*)+ w\mathbf{l}_i \geq p_i(\mathbf{T}^*)$ for every $(\mathbf{a}_i, l_i) \in F_i(\mathbf{T}^*, \mathbf{y}^*)$, and

(iv) $\mathbf{y}^* \succsim_{G(\mathbf{T}^*,\mathbf{y}^*)} \mathbf{y}$ for every $\mathbf{y} \in H(\mathbf{T}^*, \mathbf{y}^*)$.

Conditions (i) and (ii) are consistency conditions: condition (i) says that the technique $\mathbf{T}^*$ must belong to the set of the technologies known when $\mathbf{T}^*$ is used, condition (ii) says that the net product $\mathbf{y}^*$ must belong to the set of those net production vectors which are feasible and on which preferences are defined. Condition (iii) means that at technique $\mathbf{T}^*$'s prices no available method pay positive extra-profits, i.e. $\mathbf{T}^*$ is a cost-minimising technique among those known, while condition (iv) means that $\mathbf{y}^*$ must be the preferred net demand vector among those feasible and on which preferences are defined.

For every i, consider a subset Y_i of X_i. A *cost-minimising technique with respect to set* $Y = \Pi_i Y_i$ (henceforth, a CMT-Y) is a technique $\mathbf{T}^* \in \Pi_i Y_i$ such that for every i: $(1+r(\mathbf{T}^*)\mathbf{a}_i\mathbf{p}(\mathbf{T}^*)+ w\mathbf{l}_i \geq p_i(\mathbf{T}^*)$ for every $(\mathbf{a}_i, l_i) \in Y_i$. In words, a CMT-$Y$ is a cost minimising technique among those available in set Y. Consider now a subset Z of the net consumption space $\Re^n_+$. A *optimal consumption configuration with respect to set* Z (henceforth, a OCC-Z) is a net consumption vector $\mathbf{y}^*$ in Z such that $y^* \succsim_Z \mathbf{y}$, for every $\mathbf{y} \in Z$. *A cost-minimising Y-Z configuration* (henceforth, a CM-YZ) is a combination ($\mathbf{T}^*$, $\mathbf{y}^*$) such that $\mathbf{T}^*$ is a CMT-Y and $\mathbf{y}^*$ is a OCC-Z. Notice that a CM-YZ need not to be a LSC as consistency conditions (i) and (ii) in the definition of a LSC are not required in the definitions of CMT-Y and OCC-Z.

The following result is standard and it means that a CMT-Y is a technique that maximises the rate of profit among the techniques in Y.

Remark 1. For every $\mathbf{T'} \in X$, a CMT-$\Pi_i F_i(\mathbf{T'})$ is a solution to the following programme: max $r(\mathbf{T})$ sub $\mathbf{T} \in \Pi_i F_i(\mathbf{T'})$.

The dynamics of the economy is modelled according to the following adaptive process:

Adaptive process (AP): We assume that at time 0 technique $\mathbf{T}_0 = (\mathbf{A}_0, \mathbf{l}_0) \in X$ is activated and net consumption vector $\mathbf{y}_0 \in Q(\mathbf{T}_0)$ is chosen. Technique $\mathbf{T}_0$ determines the price vector $\mathbf{P}_0 = (\mathbf{p}(\mathbf{T}_0), r(\mathbf{T}_0))$ and, together with the net demand $\mathbf{y}_0$, the gross production vector $\mathbf{x}_0(\mathbf{T}_0, \mathbf{y}_0)$, according to the rules specified in the preceding section. At time 0, industry i will "discover" the set of processes $F_i(\mathbf{T}_0, \mathbf{y}_0) = \Phi_i(\mathbf{T}_0, \mathbf{x}(\mathbf{T}_0, \mathbf{y}_0), \mathbf{P}(\mathbf{T}_0))$ and these processes will be available at time 1. At the same time, households will "discover" set $G(\mathbf{T}_0, \mathbf{y}_0) = \Gamma(\mathbf{y}_0, \mathbf{P}(\mathbf{T}_0))$ of net consumption vectors, in the sense that at time 1 they are able to express their preferences on the consumption bundles in this set. It is assumed also that at time 1 a CMT-Π_i $F_i(\mathbf{T}_0, \mathbf{y}_0)$ technique is adopted, say $\mathbf{T}_1$, hence a price vector $\mathbf{P}_1 = (p(\mathbf{T}_1), \mathbf{l}(\mathbf{T}_1))$ will rule and that a OCC-$H(\mathbf{T}_0, \mathbf{y}_0)$ will be chosen, say $\mathbf{y}_1$. And so on.

Lemma 6. Under Assumptions 1-5, the **AP** is well defined for every t = 0, 1, 2, ….. Moreover, it generates a sequence of positive price vectors $\{\mathbf{P}(\mathbf{T}_t)\}$ such that for every t, if $\mathbf{T}_t \neq \mathbf{T}_{t+1}$ one obtains that $r(\mathbf{T}_{t+1}) > r(\mathbf{T}_t)$.

Proof. Assumptions 4 and 5 and Lemma 4 ensure that at each period the set upon which firms and household have to choose, i.e. $F_i(\mathbf{T}_t, \mathbf{y}_t)$ and $H(\mathbf{T}_t, \mathbf{y}_t)$, are compact. Hence, by Remark 1 and given the assumptions on households' preferences, at any period t there exists a CMT-Π_i $F_i(\mathbf{T}_{t-1}, \mathbf{y}_{t-1})$ and a OCC- $H(\mathbf{T}_{t-1}, \mathbf{y}_{t-1})$. The properties of the price sequence follow from Lemmas 1 and 2.

Proposition 1. Under Assumptions 1-5 and for whatever initial technique $\mathbf{T}_0 \in X$ and net production vector $\mathbf{y}_0 \in Q(\mathbf{T}_0)$, the **AP** stops either in a finite or in an infinite number of steps. In the former case the (finite) sequence converges to a LSC; in the latter the (infinite) sequence either converges to a LSC or the limit of every converging subsequence is a LSC.

Proof. Set $\Omega = \{(\mathbf{T}, \mathbf{y}) \in X \times \Re^n_+ \mid \mathbf{y} \in H(\mathbf{T}, \mathbf{y}), \mathbf{T} \in F(\mathbf{T}, \mathbf{y}), (1+r(\mathbf{T}))\mathbf{A}'\mathbf{p}(\mathbf{T}) + w\mathbf{l}' \geq \mathbf{p}(\mathbf{T})$ where $(\mathbf{A}', \mathbf{l}') \in F(\mathbf{T}, \mathbf{y})$, $\mathbf{y} \succsim_{H(\mathbf{T},\mathbf{y})} \mathbf{y}'$ where $\mathbf{y} \in H(\mathbf{T}, \mathbf{y})\}$. In words, Ω is the set of LCSs. By Lemma 4, set $Y = \{\mathbf{y} \in R^m \mid \mathbf{y} \in Q(\mathbf{T}), \mathbf{T} \in X\} = Q(X)$ is compact. Hence, without any loss, the elements $(\mathbf{T}, \mathbf{y})$ can be considered to belong to the compact set $X \times Y$. By the assumption on preferences and on correspondences F_i and G and from Remark 1 it follows that Ω is a closed subset of the compact set $X \times Y$, hence Ω is compact as well.

Consider now the algorithm: $\mathcal{A}$: $X \times Y \rightarrow\rightarrow X \times Y$ defined by the rule: $\mathcal{A}(\mathbf{T}', \mathbf{y}') = \{(\mathbf{T}, \mathbf{y}) \in F((\mathbf{T}', \mathbf{y}') \times H(\mathbf{T}', \mathbf{y}') \mid r(\mathbf{T}) \geq r(\mathbf{T}^{o})$, for $\mathbf{T}^{o} \in F(\mathbf{T}', \mathbf{y}')$ and $\mathbf{y} \succsim_{H(\mathbf{T}',\mathbf{y}')} \mathbf{y}^{o}$ for $\mathbf{y}^{o} \in H(\mathbf{T}', \mathbf{y}')\}$. By Remark 1, the algorithm $\mathcal{A}$ associates to each configuration $(\mathbf{T}', \mathbf{y}')$ the set of CM-$F(\mathbf{T}',\mathbf{y}')H(\mathbf{T}',\mathbf{y}')$s.

Notice that by Lemma 5 correspondences F and G are continuous. As G and Q are continuous (see the proof of Lemma 4), one has that also $H = G \cap Q$ is continuous (see e.g. Border (1995)). Thus, correspondence $\mathcal{A}$ is upper hemi continuous by Berge Theorem (see Berge (1966)) as the rate of profit is a continuous function of $\mathbf{T}$ (see Lemma 1) and under Assumption 3 households preferences can be represented by a continuous utility function (see Debreu (1954)).

Notice now that, by the behavioural assumptions on producers and consumers, the sequence of techniques and net consumption vectors generated by the **AP** can be considered as generated by the algorithm defined by correspondence $\mathcal{A}$ (by possibly making an appropriate selection of techniques and net production vectors at each period) having set Ω as solution set and with value function r (see Zangwill, 1969, Chapter 4). By what has been said before, all point $\{\mathbf{T}_t, \mathbf{y}_t\}$ belong to the compact set $X \times Y$. Hence the (infinite) sequence has at least one limit point.

Notice that if $(\mathbf{T}, \mathbf{y}) \notin \Omega$, then either there exists $\mathbf{T}^{o} = (\mathbf{A}^{o}, \mathbf{l}^{o}) \in F(\mathbf{T}, \mathbf{y})$ such that $r(\mathbf{T}^{o}) > r(\mathbf{T})$ (see Lemma 2), or there exists $\mathbf{y}^{o} \in H(\mathbf{T}, \mathbf{y})$ such that $\mathbf{y}^{o} \succ_{H(\mathbf{T},\mathbf{y})} \mathbf{y}$, while if $(\mathbf{T}, \mathbf{y}) \in \Omega$, then $r(\mathbf{T}) \geq r(\mathbf{T}^{o})$

for every $\mathbf{T}^{o} \in F(\mathbf{T}, \mathbf{y})$ and $\mathbf{y} \succ_{H(\mathbf{T},\mathbf{y})} \mathbf{y}^{o}$ for every $\mathbf{y}^{o} \in H(\mathbf{T}, \mathbf{y})$. Therefore, by Zangwill Convergence Theorem A (see Zangwill, 1969, p. 91), the limit of any convergent subsequence of $\{\mathbf{T}_t, \mathbf{y}_t\}$, say $(\mathbf{T}^*, \mathbf{y}^*)$, belongs to Ω.

The result above does not exclude that the (infinite) sequence generated by the algorithm has more than one limit point. From the empirical point of view this feature of our model is quite disturbing, as this would mean that according our model the economies can "jump" permanently around more than one technique or net consumption vector. This case would contradict the literature on "technological trajectories" (see, e.g. Dosi (1982)), which is strongly supported by empirical evidence. In order to avoid this phenomenon, therefore, we introduce three conditions each of them ensuring that the whole sequence generated by the AP converges towards a LSC.

Assumption 6(i). There is only one LCS.

Under Assumption 6(i) it is trivial to see that the whole sequence converges to the LCS. His assumption, however, is not very interesting from the economic point of view.

Assumption 6(ii). There exists a family of disjoint compact neighbourhoods of the LCS, say Δ, such that each element of the family contains at most one LSC and, if $(\mathbf{T}, \mathbf{y})$ is a LCS and $(\mathbf{T}, \mathbf{y}) \in N(\mathbf{T}, \mathbf{y}) \in \Delta$, then $\prod_i F_i(\mathbf{T}, \mathbf{y}) \times G(\mathbf{T}, \mathbf{y}) \subset N(\mathbf{T}, \mathbf{y})$.

Assumption 6(ii) has a very clear economic meaning: it means that from a secular equilibrium is not possible to "jump" in one step to another secular equilibrium. If we interpret secular equilibria as the outcome of a long period of knowledge development defining technological and preferences trajectories, then such assumption amounts to saying that it is not possible at the end of a technological and preferences trajectory to jump in one "period" to the end of an alternative trajectory. Hence, Assumption 6(ii) amounts to emphasising the cumulative nature of knowledge and the time dimension of knowledge as far as technology and preferences are concerned.

Proposition 2. Under Assumptions 1-5 and 6(ii) the (whole infinite) sequence $\{\mathbf{T}_t, \mathbf{y}_t\}$ generated by the AP converges to a LCS.[4]

Proof. First we show that if $\{\mathbf{T}_t, \mathbf{y}_t\}$ is a sequence generated by the AP, then $d(\mathbf{T}_t, \mathbf{T}_{t+1}) \to 0$ and $d(\mathbf{y}_t, \mathbf{y}_{t+1}) \to 0$ as $t \to \infty$, where d is any metric defined on X and R^n. Suppose not. Then, there exist a sub-sequence $\{\mathbf{T}_t, \mathbf{y}_t\}$ such that either $d(\mathbf{T}_t, \mathbf{T}_{t+1}) \to \beta > 0$ or $d(\mathbf{y}_t, \mathbf{y}_{t+1}) \to \beta > 0$ as $t \to \infty$. Without loss of generality we can assume that $d(\mathbf{T}_t, \mathbf{T}_{t+1}) \to \beta > 0$ as $t \to \infty$. We may assume that $\mathbf{T}_t$ converges to $\mathbf{T}^*$ and $\mathbf{T}_{t+1}$ converges to $\mathbf{T}^{**}$. Clearly, $d(\mathbf{T}^*, \mathbf{T}^{**}) \geq \beta$. By Proposition 1, $\mathbf{T}^*$ and $\mathbf{T}^{**}$ are LSCs, moreover $\mathbf{T}^{**} \in F(\mathbf{T}^*)$ (because $\mathbf{T}_{t+1} \in F(\mathbf{T}_t)$ for every t), by this contradicts Assumption 6(iii).

Suppose now that the assertion is not true. Therefore, if $\{\mathbf{T}_t, \mathbf{y}_t\}$ is a sequence generated by the AP, then there must exist at least two subsequences, say $\{\mathbf{T}_{t'}, \mathbf{y}_{t'}\}$ and $\{\mathbf{T}_{t''}, \mathbf{y}_{t''}\}$ converging to $\{\mathbf{T}', \mathbf{y}'\}$ and $\{\mathbf{T}'', \mathbf{y}''\}$, respectively. By Proposition 1 again, every accumulation point of sequence $\{\mathbf{T}_t, \mathbf{y}_t\}$ is a LSC, therefore, by Assumption 6(iii) it is possible to take two positive numbers ε and ϕ such that $(\mathbf{T}', \mathbf{y}')$ and $(\mathbf{T}'', \mathbf{y}'')$ are the sole accumulation points in $B_\varepsilon(\mathbf{T}_{t'}) \times B_\varepsilon(\mathbf{y}_{t'})$, and $B_f(\mathbf{T}_{t''}) \times B_f(\mathbf{y}_{t''})$, respectively, where $B_\varepsilon(\mathbf{T}') \subset N(\mathbf{T}')$, $B_\varepsilon(\mathbf{y}') \subset N(\mathbf{y}')$, $B_\phi(\mathbf{T}'') \subset N(\mathbf{T}'')$, $B_\phi(\mathbf{y}'') \subset N(\mathbf{y}'')$. Choose any positive number Z in such a way that min[$d(\mathbf{T}_z,\mathbf{T}_{z+1})$, $d(\mathbf{y}_z,\mathbf{y}_{z+1})$] $< \varepsilon/3$ for $z \geq Z$ (That such a number exists follows from the result at the beginning of this proof). However, $(\mathbf{T}', \mathbf{y}')$ is an accumulation point of sequence $\{\mathbf{T}_t, \mathbf{y}_t\}$, therefore, for infinitely many indices s one has that $d(\mathbf{T}_s, \mathbf{T}') < \varepsilon/3$ and $d(\mathbf{y}_s, \mathbf{y}') < \varepsilon/3$. On the other hand, $(\mathbf{T}'', \mathbf{y}'')$ is another accumulation point of sequence $\{\mathbf{T}_t, \mathbf{y}_t\}$, hence, by the fact that $(\mathbf{T}'', \mathbf{y}'') \notin B_\varepsilon(\mathbf{T}_{t'}) \times B_\varepsilon(\mathbf{y}_{t'})$, there must exist infinitely many indices $m \geq Z$ such that $(\varepsilon/3) \leq d(\mathbf{T}_m, \mathbf{T}') \leq (2\varepsilon/3)$ and $(\varepsilon/3) \leq d(\mathbf{y}_m, \mathbf{y}') \leq (2\varepsilon/3)$. This

[4] It may be worth emphasising the following methodological fact: the standard dynamic theory puts emphasis upon recurrent points (equilibria) and considers non recurrent points as being not very important. Thus, the state space is studied only in terms of recurrent points (for a clear statement of this methodological position see Akin (1993 p. 2)). By contrast, in our analysis, we adopt the opposite viewpoint: our attention is mainly on the behaviour of transitory states and we introduce assumptions upon the recurrent points in order to obtain a “nice” dynamics from the economic point of view.

implies that there exists an accumulation point in the set $\{\mathbf{T}\in X|\ (\varepsilon'/3) \leq d(\mathbf{T}, \mathbf{T}') \leq (2\varepsilon'/3)\}\times\{\mathbf{y}\in X|\ (\varepsilon'/3) \leq d(\mathbf{T}, \mathbf{T}') \leq (2\varepsilon'/3)\} \subset N(\mathbf{T}') \times N(\mathbf{y}')$, which contradicts Assumption 6(ii).

Notice that in the preceding proof the crucial step is showing that the distance of successive elements of the sequence tends to zero as t tends to infinity. This condition can be ensured in a more straightforward way, which turns out to be also much more appealing from the economic point of view, i.e. to assume that the size of the correspondences F and G decreases over time and collapse asymptotically into a point. This assumption essentially means from the economic point of view that there are decreasing returns in discovering new technology and that the exploration by the households of the space of net production in order to define their preferences tends to exhaust over time. These assumptions are quite palatable and, at least for the technological side, is known as Wolf's Law and it is widely assumed (see e.g. Young (1993, f. 3)) and empirically verified. So, by setting $\mathrm{diam}(X) = \inf\{d\in R|\ X \subset B_d\}$ where X is a generic set in R^n and B_d is the ball of radius d in R^n, we have:

Assumption 6(iii). For every sequence $\{(\mathbf{T}_t, \mathbf{y}_t)\}_t$ and for every i, $\mathrm{diam}(F_i(\mathbf{T}_t, \mathbf{y}_t)) \to 0$ and $\mathrm{diam}(G(\mathbf{T}_t, \mathbf{y}_t)) \to 0$ as $t\to\infty$.

The proof of the following result is easy and similar to the second part of the Proof of Proposition 2.

Proposition 3. Under Assumptions 1-5 and 6(iii) any (infinite) sequence $\{(\mathbf{T}_t, \mathbf{y}_t)\}_t$ generated by the AP converges to a LSC.

4. Structural change and product innovation

As anticipated in the Introduction, one of the main features of modern economies is the ever changing number of commodities produced. Such a phenomenon is moreover considered as one of the main drive of a sustained growth of economies over time. Unfortunately, the existing models of structural change does not seem to be able to tackle such a phenomenon as either they explicit rule out change in the number of product or they allow it but in an exogenously way. Thus, one may rightly wonder whether the model developed in the preceding section is able to deal with a changing number of commodities over time. In this section we shall provide two numerical examples showing that the answer is positive. In the first example, we shall illustrate the case of emergence of new pure consumption goods, while in the second example we shall illustrate the case of the emergence of new pure capital goods. More complex cases can be easily envisaged by the reader.

Example 1. Emergence of consumption goods

Consider a two good economy, call them "corn", indicated by *c,* and "tea" indicated by *t*. Suppose that there exists only one method to produce each of the two commodities and that there is no technical progress. Suppose also that the wage rate is equal to 0.1 and that corn is used as *numeraire*. Let $a_c^{\mathrm{o}} = \left(a_{cc}^{\mathrm{o}}, a_{ct}^{\mathrm{o}}, l_c^{\mathrm{o}}\right) = (0.1, 0, 1)$ be the method to produce corn and $a_t^{\mathrm{o}} = (a_{tc}^{\mathrm{o}}, a_{tt}^{\mathrm{o}}, l_t^{\mathrm{o}}) = (0.1, 0, 1.1)$ the method to produce tea. Suppose also that labour is available in unlimited amount, so any vector in the non negative orthant is feasible. Finally, assume that (aggregate) preferences of consumers are given over the space of net product and they are represented by the utility function: $U(y_c, y_t) = (y_c+1)(y_t+1)-1$, and that the law of motion defining the exploration of the net demand space is:

$$G(y^o) = \left\{ y \in \Re_+^2 \middle| y = y^{\mathrm{o}}, y = y^{\mathrm{o}} + \varepsilon, y = y^{\mathrm{o}} + \max\left[0, |y| - K\right] e(y), y = y^{\mathrm{o}} + \varepsilon + \max\left[0, |y| - K\right] e(y) \right\}$$

where $y^{\mathrm{o}} \in \Re_+^2$ is the status quo, ε and K are positive real numbers, $|\cdot|$ indicates the norm operator and $e(y^{\mathrm{o}}) \in H^{\perp}(y^{\mathrm{o}}) = \left\{ y \in \Re_+^2 \middle\| y| = 1, y \cdot y^{\mathrm{o}} = 0 \right\}$.

Figure 1 illustrates set $G(y^{\mathrm{o}})$ for the case $y^{\mathrm{o}} = (1, 1)$, $\varepsilon = \frac{\sqrt{2}}{2}$ and $K = 1$. This set is given by the six vectors y^{o}, $y^{\mathrm{o}} + \varepsilon$, a, b, c and d, where the distance between y^{o} and, for example, d is equal to $\sqrt{2} - 1$.

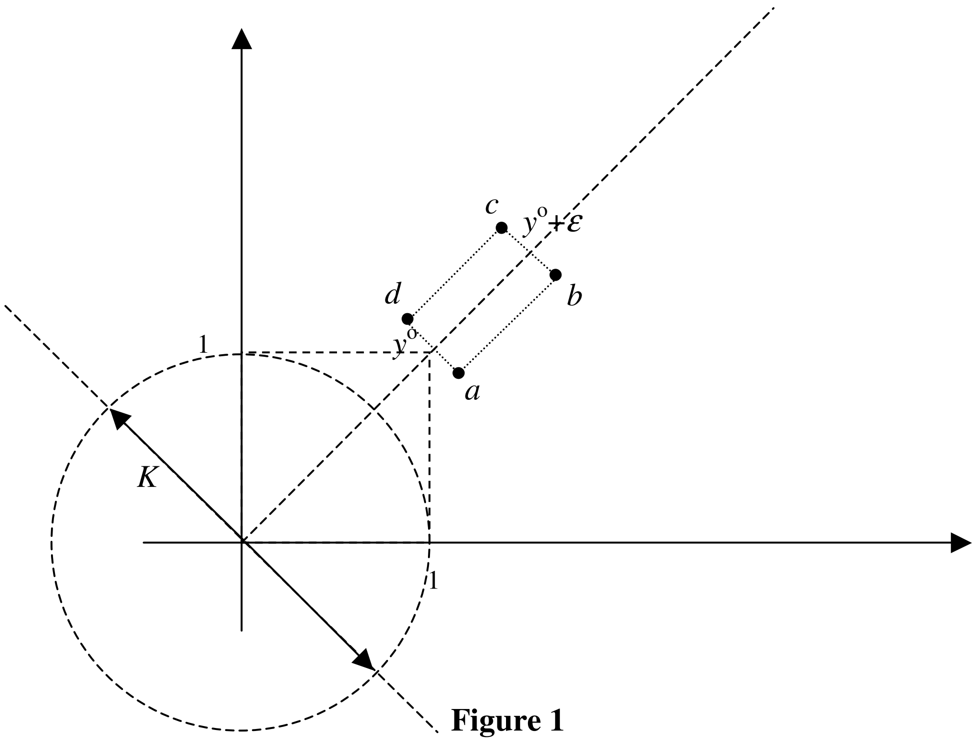

Figure 1

Now, set $y^{o} = (y_c^{o}, 0)$ with $y_c^{o} < 1$. It is immediate to check that for any period $t \leq t^*$, $G(y^t) = \left\{(y_c, y_t) \in \Re_+^2 \,\middle|\, y = y_c^t, y_c = y_c^t + \varepsilon, y_t^t = 0\right\}$, where $t^* = \min\left\{t \in \mathbb{N} \,\middle|\, t > \frac{K - y^{o}}{\varepsilon}\right\}$. It follows that from time 0 to time t^*-1 only corn is demanded and produced. However, from $t = t^*$ onwards, $|y_t| > K$ and $\max\left[0, |y_t| - K\right] = |y_t| - K > 0$ hence, $G(y^t) = \left\{y \in \Re_+^2 \,\middle|\, y = y^t, y = y^t + \varepsilon, y = y^t + \max\left[0, |y| - K\right] e^+(y), y = y^t + \varepsilon + \max\left[0, |y| - K\right] e^+(y)\right\}$, where $e^+(y^t) = (0,1)$ (see Figure 2). Obviously, by strict monotonicity, it follows that bundle $(y_c^t + \varepsilon, |y| - K)$ will be chosen. Thus, while up to time t^*-1 only corn is demanded, from time t^* onwards corn and tea are demanded and produced.

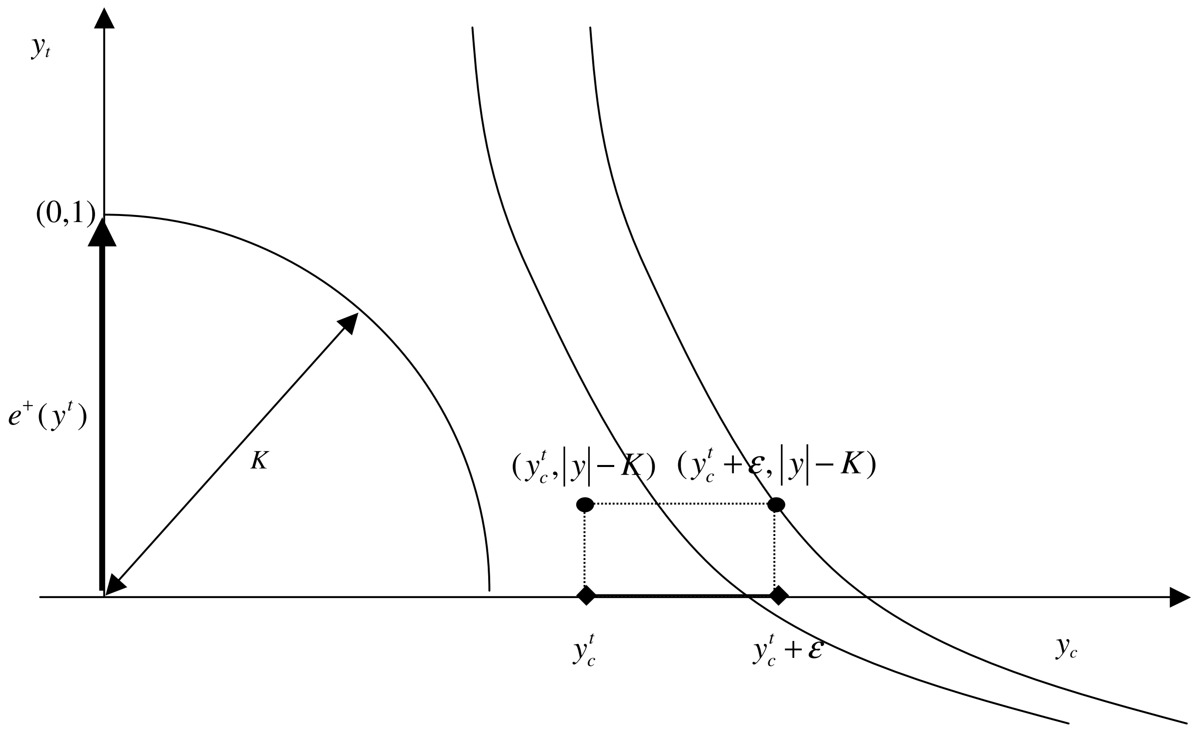

Figure 3

Example 2. Emergence of capital goods.

Let us consider an economy in which only two commodities can potentially be produced, called "corn" and "fertiliser". Corn is not only a consumption good but it is also used as circulating capital, while fertiliser can be used only as circulating capital. Either commodities are produced by means of constant returns to scale methods of production. More specifically, fertiliser could be produced via the following method of production $a_f = (a_{fc}, a_{ff}, l_f) = (0.1, 0, 1)$, where a_{fc} indicates the amount of corn used to produce one unit of fertiliser, a_{ff} indicates the amount of fertiliser used to produce one unit of fertiliser and l_f indicates the amount of labour necessary to produce one unit of fertiliser. It is assumed that the method used in the fertiliser industry does not change over time. By contrast, the corn industry is characterised by a changing set of available methods according to the following rule (with obvious meaning):

$$F(a_c^{\mathrm{o}})=\left\{a_1(a_c^{\mathrm{o}})=\begin{pmatrix}a_{cc}^{\mathrm{o}}\\ a_{cf}^{\mathrm{o}}\\ l_c^{\mathrm{o}}\end{pmatrix},a_2(a_c^{\mathrm{o}})=\begin{pmatrix}a_{cc}^{\mathrm{o}}\\ a_{cf}^{\mathrm{o}}\\ l_c^{\mathrm{o}}-0.1\end{pmatrix},a_3(a_c^{\mathrm{o}})=\begin{pmatrix}a_{cc}^{\mathrm{o}}-0.01\\ a_{cf}^{\mathrm{o}}+0.01\\ \max\left[0.001,l_c^{\mathrm{o}}-4(1-l_c^{\mathrm{o}})\right]\end{pmatrix}\right\},\quad (*)$$

where $a_c^{\mathrm{o}}=(a_{cc}^{\mathrm{o}},a_{cf}^{\mathrm{o}},l_c^{\mathrm{o}})$ indicates the currently used method. Rule (*) indicates that each current technique in producing corn, a_c^{o}, makes available three other methods, indicated by $a_1(a_c^{\mathrm{o}})$, $a_2(a_c^{\mathrm{o}})$ and $a_3(a_c^{\mathrm{o}})$ with the features indicated by the above rule.

Suppose that the wage rate is given and equal to one, and that corn is used as *numeraire*. Finally, suppose that at time 0 there is only one available method to produce corn and it is the following: $a_c^{\mathrm{o}}=(a_{cc}^{\mathrm{o}},a_{cf}^{\mathrm{o}},l_c)=(0.1,0,1.1)$. From rule (*) it follows that the following three methods will be available in the economy at time 1:

$a_1(a_c^{\mathrm{o}})=(0.1,0,1.1)$, $a_2(a_c^{\mathrm{o}})=(0.1,0,1)$, $a_3(a_c^{\mathrm{o}})=(0.09,0.01,1.5)$.

It can be checked that the cost-minimising technique is $a_2(a_c^{\mathrm{o}})$, i.e. fertiliser is neither produced nor used up in production. As a matter of fact, this technique yields a profit rate equal to 8, while the technique made up by $a_3(a_c^{\mathrm{o}})$ and a_f yields a profit rate equal to 7.5393. Considering $a_c^1=a_2(a_c^{\mathrm{o}})$ as status quo technique at time 1, rule (*) implies that at time 2 the following three methods will be available to produce corn:

$a_1(a_c^1)=(0.1,0,1)$, $a_2(a_c^1)=(0.1,0,0.9)$, $a_3(a_c^1)=(0.09,0.01,1)$.

Also in this case, it is possible to check that the cost-minimising technique is made only by method $a_2(a_c^1)$, which implies that fertiliser is neither produced nor used to produce corn. As a matter of fact, this technique yields a profit rate equal to 8.1, while the technique made up by $a_3(a_c^{\mathrm{o}})$ and a_f yields a profit rate equal to 8. Considering $a_c^2=a_2(a_c^1)$ as status quo technique at time 2, rule (*) implies that at time 3 the following three methods will be available to produce corn:

$a_1(a_c^2)=(0.1,0,0.9)$, $a_2(a_c^2)=(0.1,0,0.8)$, $a_3(a_c^2)=(0.09,0.01,0.5)$.

Unlike the preceding period, at time 3 the cost minimising technique is given by methods $a_3(a_c^2)$ and a_f, as it yields a profit rate equal to 8.4568, while the technique made only by method $a_2(a_c^2)$ yields a profit rate equal to 8.2. It follows that while in periods 0, 1 and 2 the economy produces only corn, at time 3 corn and fertiliser are produced.

5. Heterogeneous firms: towards an evolutionary general equilibrium analysis of structural change

The model developed in the preceding sections is based on the idea that agents (firms and consumers) decide according to their knowledge of the relevant parameters and assumes that each industry can be represented by only one process of production. The latter assumption can be justified on several grounds. The usual interpretation used by the follower of the "classical" approach (see, e.g. Kurz and Salvadori (1995), Pasinetti (1975)) is that as production prices represent long period prices, then it is natural to assume that all firms are able to use *the* best technology. However, empirical works does not seem to support such a conclusion as firms operating in the same industry persistently maintain different methods of production, with different levels of efficiency (see, e.g. Bartelsman and Doms (2000) and Nelson (1991)). Particular emphasis has been put on the persistence of differential productivity by the evolutionist literature, which explains the origin of such differences on the basis of the existence of tacit knowledge which makes each firm unique in its knowledge assets. This interpretation fits well with our approach as, having made explicit the knowledge set, we can assume that the initial technology knowledge set of each firm is different and that each firm experiences a different evolution of this set. In our approach, therefore, persistent heterogeneity of firms is particularly appropriate.

If we extend the analysis to this direction we are able to extend the study of evolution of industries within a partial equilibrium approach as carried out for example by Metcalfe (1995, 1998), Metcalfe, Fonseca and Ramlogan (2001), Montobbio (2002) to a truly multisectoral linear production model with structural dynamics. Therefore, our model could represent the bridge between Pasinetti's model of structural change and the literature on evolutionary economics.

To this end, we assume that in sector i there are m_i firms and let us indicate by j_{fi} the generic j-th process available to firm f. The process j-th available to firm f in sector i is represented by the couple $\mathbf{b}_{jfi}= (\mathbf{a}_{jfi}, l_{jfi})$. Set $M_i = \{1, 2, \ldots, m_i\}$. Let us assume for the sake of simplicity that at time 0 there is only one process available to each firm, denoted process 1, and denote by $\mathbf{b}_{1fi0}= (\mathbf{a}_{1fi0}, l_{1fi0})$ the process available to firm f in sector i at time 0. We call *elementary technique* any technique which is made up by only one method in each industry. Therefore, there are m_1x m_2x….x m_n elementary techniques. Let $\mathbf{T}_0 = (\mathbf{A}_0, \mathbf{l}_0)$ be the technique made up by the processes defining all firms in the economy and $\mathbf{T}^{\mathbf{f}0} = (\mathbf{A}^{\mathbf{f}0}, \mathbf{l}^{\mathbf{f}0})$ be a generic elementary technique made up by firm f_i in sector i, where $\mathbf{f}_0 = (f_1, f_2, \ldots f_n)$. Indicate by C_0 the set of indices $\mathbf{f}_0$ of elementary techniques obtained considering in each sector all possible combinations of firms. Assuming that for each elementary technique the usual technological conditions are satisfied (see Assumptions 1 and 2), it follows that each elementary technique yields a positive price vector $\mathbf{p}^{\mathbf{f}0}$ and a positive rate of profit $r^{\mathbf{f}0}$. Choose any elementary technique yielding the *lowest* rate of profit, say $\mathbf{f'}_0 = (f'_1, f'_2, \ldots f'_n)$.; i.e. $r^{\mathbf{f'}0} \leq r^{\mathbf{f}0}$ for every $\mathbf{f}_0 \in C_0$ (the existence of this technique can be easily shown). Let $\mathbf{p}^{\mathbf{f'}0}$ be the associated price vector

Suppose now that *all* m_i firms are operating in sector i at price $\mathbf{p}^{\mathbf{f'}0}$ and rate of profit $r^{\mathbf{f'}0}$. It is easy to show that any other firm $f_{i0} \neq f'_{i0}$ yields non negative extra profits, γ^{fi0}; i.e. γ^{fi0} satisfies the equality:[5] $\gamma^{fi0} + (1+ r^{\mathbf{f'}0})\mathbf{a}_{1fi}\mathbf{p}^{\mathbf{f}0} + wl_{1fi} = p_i^0$. Indicate by γ_{i0} the average extra profits in sector i at time 0; i.e.

$$\gamma_{i0} = \sum_{fi} \gamma^{fi0} \Big/ m_i \,.$$

As for the "quantity side" we consider the "aggregate" technology $\mathbf{T}_0(\mathbf{s})$ defined as the technology obtained as a linear combination of the processes in each sector weighted by the market share of firms and where $\mathbf{s}$ indicates a vector with $m_1+m_2+....+ m_n$ elements each of which is the market share of each firm in each industry. For the sake of simplicity, we assume that at time 0 the share of each firm in producing the gross production of good i is equal for all firms to the number $s_i = 1/m_i$. Thus $\mathbf{T}_0(\mathbf{s}) = (\mathbf{A}_0(\mathbf{s}), \mathbf{l}_0(\mathbf{s}))$ where the process $\mathbf{b}_0(\mathbf{s})$ in industry i in matrix $\mathbf{A}_0(\mathbf{s})$ is $\mathbf{b}_0(\mathbf{s}) = (\mathbf{a}_0(\mathbf{s}), l_0(\mathbf{s})) = (\Sigma_{fi} s_i \mathbf{a}_{1fi0}\ , \Sigma_{fi} s_i l_{1fi0}\ )$. The aggregate technique $\mathbf{T}_0$ determines the gross production vector $\mathbf{x}_0$, given the net production vector $\mathbf{y}_0$; i.e. vector $\mathbf{x}_0$ is the solution to the equation $\mathbf{x}_0 = \mathbf{A}_0(\mathbf{s})\ \mathbf{x}_0 + \mathbf{y}_0$. Thus, at period 0 the generic firm f_i in sector i will produce $s_i x_i(0)$. We assume also that at period 1 the market share of each firm depends upon the difference between its extraprofits and the average one in the same industry at time 0; i.e. $s_{if} = \max\ [0, s_i + h_i(\gamma^{f0} - \gamma_{i0})]$, where $h_i > 0$. Moreover, at period 1 firm f_i in industry i will learn a new set of techniques $F_{fi}(\mathbf{T}_0, \mathbf{y}_0)$ and consumers will define their preferences of the subset $G(\mathbf{T}^{\mathbf{f}'0}, \mathbf{y}_0)$. Denote by P_{fi1} the set of indices of the processes available to firm f in industry i at time 1 and by $\mathbf{b}_{jfi1} = (\mathbf{a}_{jfi1}, l_{jfi1})$ the generic process j available to firm f in sector i at time 0.

A generic elementary technique at time 1 is indicated by $\mathbf{T}^{\mathbf{fj}1}$. Again, consider the set of hypothetical elementary techniques, and denote by $\mathbf{T}^{(\mathbf{j},\ \mathbf{f})1} = (\mathbf{A}^{(\mathbf{j},\ \mathbf{f})1}, \mathbf{l}^{(\mathbf{j},\ \mathbf{f})1})$ be a generic elementary technique made up by process j_i operated by firm f in sector i, where $(\mathbf{j}, \mathbf{f}) = (j_1 f_1, j_2 f_2, \ldots, j_n f_n)$, $j_i \in P_{fi1}$, $f_i = 1, 2, \ldots, m_i$. If all possible elementary techniques satisfy the appropriate technological conditions for ensuring positive price vector $\mathbf{p}^{(\mathbf{j},\ \mathbf{f})1}$ and positive uniform rate of profit $r^{(\mathbf{j},\ \mathbf{f})1}$ (see Assumptions 1 and 2), we can define the uniform rate of profit of this economy r^{*1} as the maximum rates of profit which will be yielded by choosing the "worst" firm in each sector (i.e. that one which yields the lowest rate of profit by choosing the technique yielding the highest rate of profit). In formal terms: $r^{*1} = \min_{\mathrm{f}}\left[\max_{\mathrm{j}} r(T^{(\mathrm{j},\ \mathrm{f})1})\right]$. The associated price vector is $\mathbf{p}^{*1}$. It is easy to see at these price vector all firms in all sectors yields non negative extra profits by choosing their best (i.e.

[5] The reader will notice the similarity of our approach with the extensive rent case in Sraffa (1960, Chapter XI).

extra-profit maximising) process. In this way we can calculate the market share of all firms in each sector next period , and so on and so forth.

Within this framework, it is possible in principle to develop the descriptive evolutionary analysis of the industries as that one in Metcalfe (1995)(1998) and in Montobbio (2003). Unlike their work, however, the analysis which can be carried out from our model has the advantage to determine endogenously the price vector on the ground of a full fledged multisectoral model of structural change.

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